

Chapter 4

PARABOLIC DISH ANTENNAS

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Introduction

Parabolic dish antennas can provide extremely high gains at microwave frequencies. A 2-foot dish at 10 GHz can provide more than 30 dB of gain. The gain is only limited by the size of the parabolic reflector; a number of hams have dishes larger than 20 feet, and occasionally a much larger commercial dish is made available for amateur operation, like the 150-foot one at the Algonquin Radio Observatory in Ontario, used by VE3ONT for the 1993 EME Contest.

These high gains are only achievable if the antennas are properly implemented, and dishes have more critical dimensions than horns and lenses. I will try to explain the fundamentals using pictures and graphics as an aid to understanding the critical areas and how to deal with them. In addition, a computer program, **HDL_ANT** is available for the difficult calculations and details, and to draw templates for small dishes in order to check the accuracy of the parabolic surface.

Background

In September 1993, I finished my 10 GHz transverter at 2 PM on the Saturday of the VHF QSO Party. After a quick checkout, I drove up Mt. Wachusett and worked four grids using a small horn antenna. However, for the 10 GHz Contest the following weekend, I wanted to have a better antenna ready.

Several moderate-sized parabolic dish reflectors were available in my garage, but lacked feeds and support structures. I had thought this would be no problem, since lots of people, both amateur and commercial, use dish antennas. After reading several articles in the ham literature, I had a fuzzy understanding and was able to put a feed horn on one of the dishes and make a number of contacts >200 km. from Mt. Washington, in horizontal rain.

However, I was not satisfied that I really understood the details of making dishes work, so I got some antenna books from the library and papers from IEEE journals and did some reading. [This book is the result of several more years of reading and writing about antennas.] The 10 GHz antenna results from the 1993 Central States VHF Conference suggest that I might not be the only one who is fuzzy on the subject — the dishes measured had efficiencies from 23% to less than 10%, while all the books say that efficiency should typically be 55%. On the other hand, there are enough hams doing

successful EME work to suggest that some have mastered feeding their dishes. One of them, VE4MA, has written two good articles^{1,2} on TVRO dishes and feedhorns for EME.

There have been some good articles written by antenna experts who are also hams, like KI4VE³, K5SXX⁴, and particularly W2IMU⁵ in *The ARRL UHF/Microwave Experimenter's Manual*, which is an excellent starting point. However, as I struggled to understand things that are probably simple and obvious to these folks, I did some reading and then used my computer to do some of the more difficult calculations and plot them in ways that helped me to understand what is happening. Many of us find a picture easier to comprehend than a complex equation. What I hope to do here is to start at a very basic level and explain the fundamentals, with pictures and graphics, well enough for hams to implement a dish antenna that works well. An accompanying computer program, **HDL_ANT**, is provided to do the necessary design calculations.

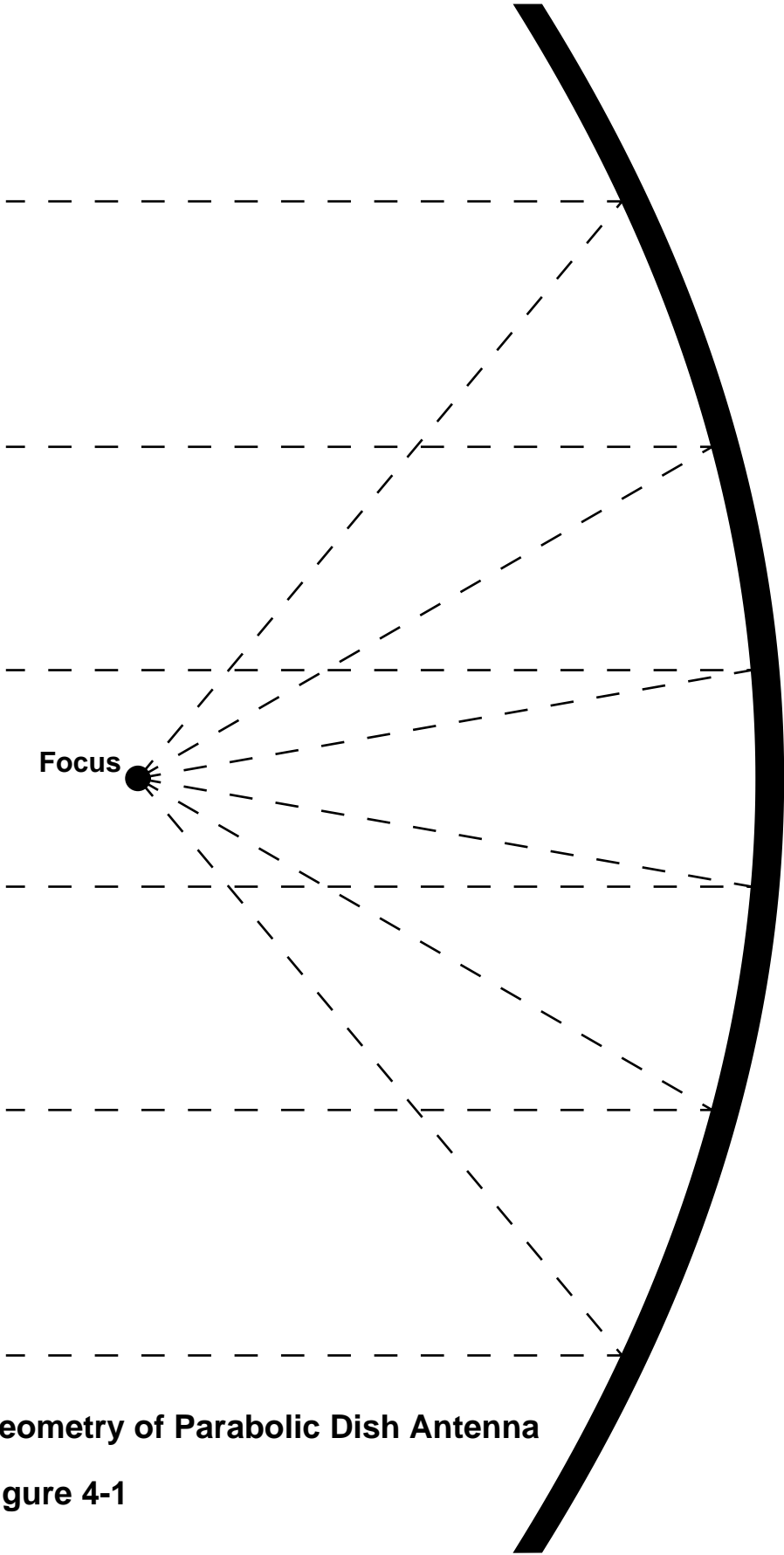
Dish Antenna Design

A dish antenna works the same way as a reflecting optical telescope. Electromagnetic waves, either light or radio, arrive on parallel paths from a distance source and are reflected by a mirror to a common point, called the focus. When a ray of light reflects from a mirror or flat surface, the angle of the path leaving (angle of reflection) is the same as the angle of the path arriving (angle of incidence). This optical principle is familiar to anyone who misspent a part of his youth at a pool table! If the mirror is a flat surface, then two rays of light leave in parallel paths; however, if the mirror is curved, two parallel incident rays leave at different angles. If the curve is parabolic ($y = ax^2$) then all the reflected rays meet at one point, as shown in Figure 4-1. A dish is a parabola of rotation, a parabolic curve rotated around an axis which passes through the focus and the center of the curve.

A transmitting antenna reverses the path: the light or radio wave originates from a point source at the focus and is reflected into a beam of rays parallel to the axis of the parabola, as shown in Figure 4-1.

Illumination

Some of the difficulties found in real antennas are easier to understand when considering a transmitting antenna, but are also present in receiving antennas, since antennas are reciprocal. One difficulty is finding a point source, since any antenna, even a half-wave dipole at 10 GHz, is much bigger than a point. Even if we were able to find a point source, it would radiate equally in all directions, so the energy that was not radiated toward the reflector would be wasted. The energy radiated from the focus toward the reflector illuminates the reflector, just as a light bulb would. So we are looking for a point source that illuminates only the reflector.



Geometry of Parabolic Dish Antenna

Figure 4-1

Aperture, Gain, and Efficiency

The aperture, gain, and efficiency of an antenna were all defined in Chapter 1 for antennas in general. The aperture A of a dish antenna is the area of the reflector as seen by a passing radio wave:

$$A = \pi r^2$$

where r is the radius, half of the diameter of the dish.

If we replace a dish antenna with a much larger one, the greater aperture of the larger is capturing much more of the passing radio wave, so larger dish has more gain than the smaller one. If we do a little geometry, we find that the gain is proportional to the aperture.

The gain of a dish is calculated as described in Chapter 1:

$$G_{dBi} = 10 \log_{10} \left(\eta \frac{4\pi}{\lambda^2} A \right)$$

with reference to an isotropic radiator; η is the efficiency of the antenna. It might be amusing to calculate the gain of the VE3ONT 150-foot dish at various frequencies; use 50% efficiency to make the first calculation simpler, then try different values to see how efficiency affects gain.

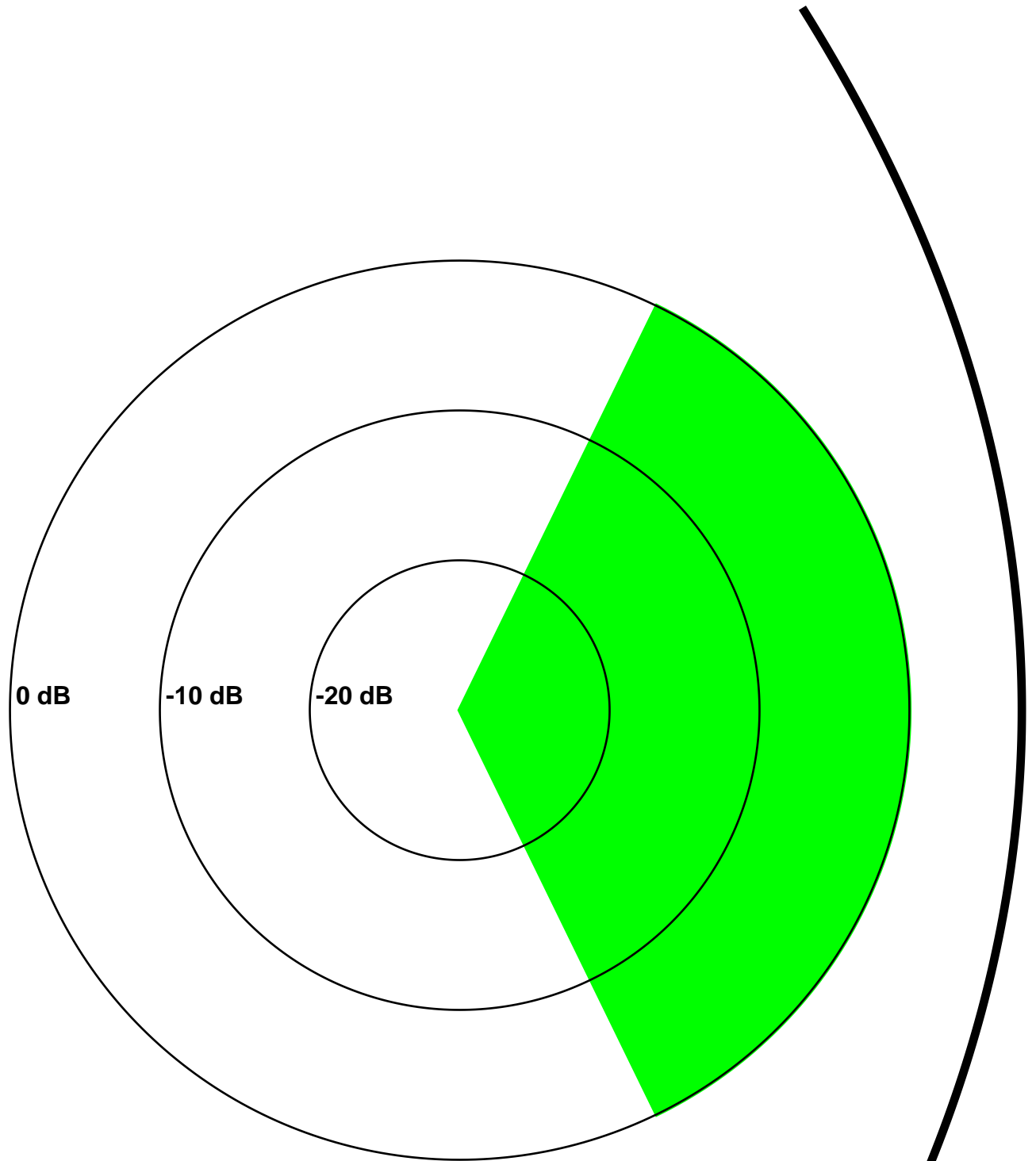
How much efficiency should we expect? All the books say that 55% is reasonable, and 70 to 80% is possible with very good feeds. Several ham articles have calculated gain based on 65% efficiency, but I haven't found measured data to support any of these numbers. On the other hand, KI4VE³ suggests that the amateur is lucky to achieve 45-50% efficiency with a small dish and a typical "coffee-can" feed.

PRACTICAL DISH ANTENNAS

When we first described a parabolic dish antenna, we put a point source at the focus, so that energy would radiate uniformly in all directions both in magnitude and phase. The problem is that the energy that is not radiated toward the reflector will be wasted. What we really want is a feed antenna that only radiates toward the reflector, and has a phase pattern that appears to radiate from a single point.

Feed Patterns

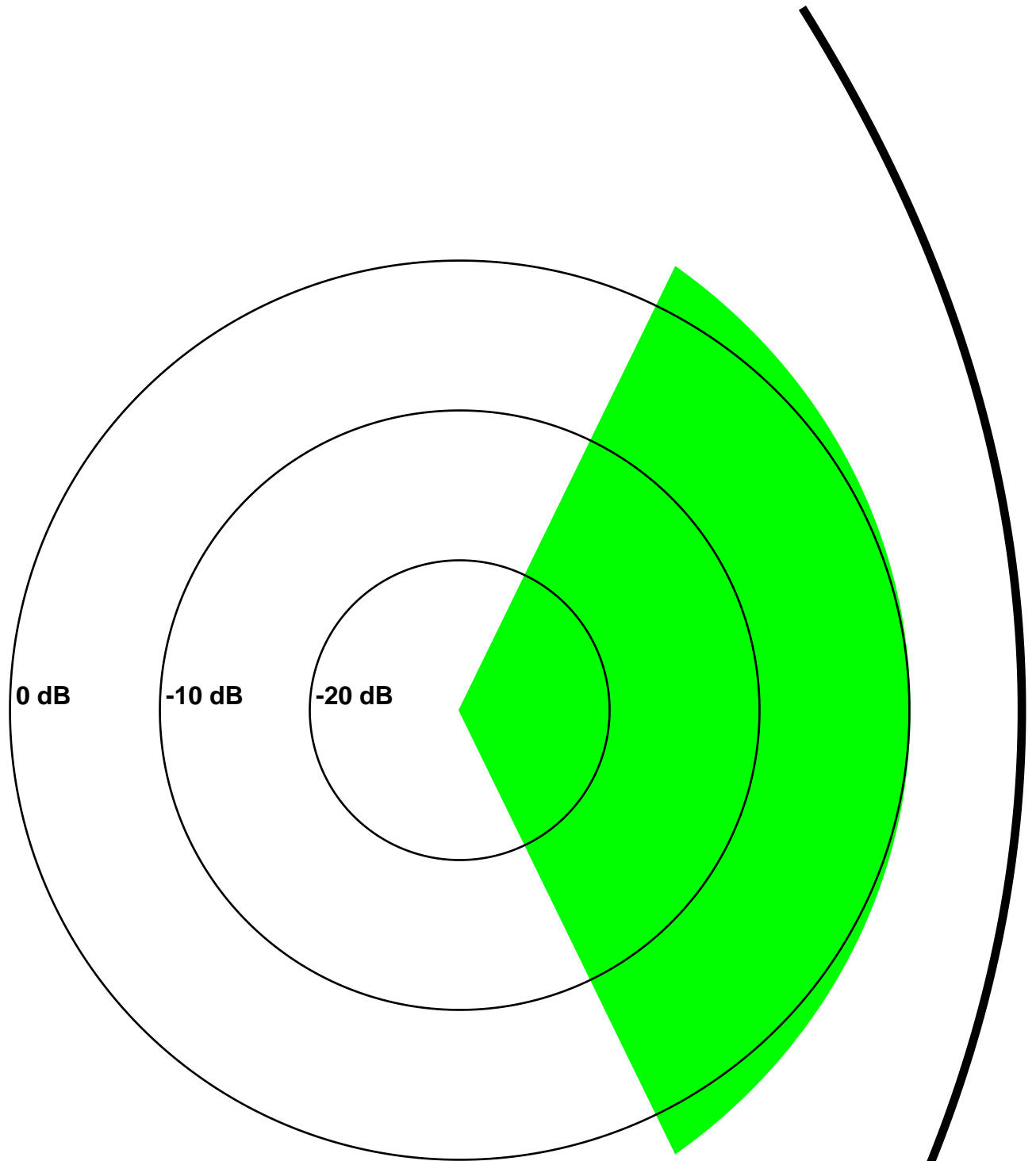
We have already seen that efficiency is a measure of how well we use the aperture. If we can illuminate the whole reflector, then we should be using the whole aperture. Perhaps our feed pattern should be as shown in Figure 4-2, with uniform feed illumination across the reflector. But when we look more closely at the parabolic surface, we find that the



Parabolic Dish Antenna with Uniform Feed Illumination

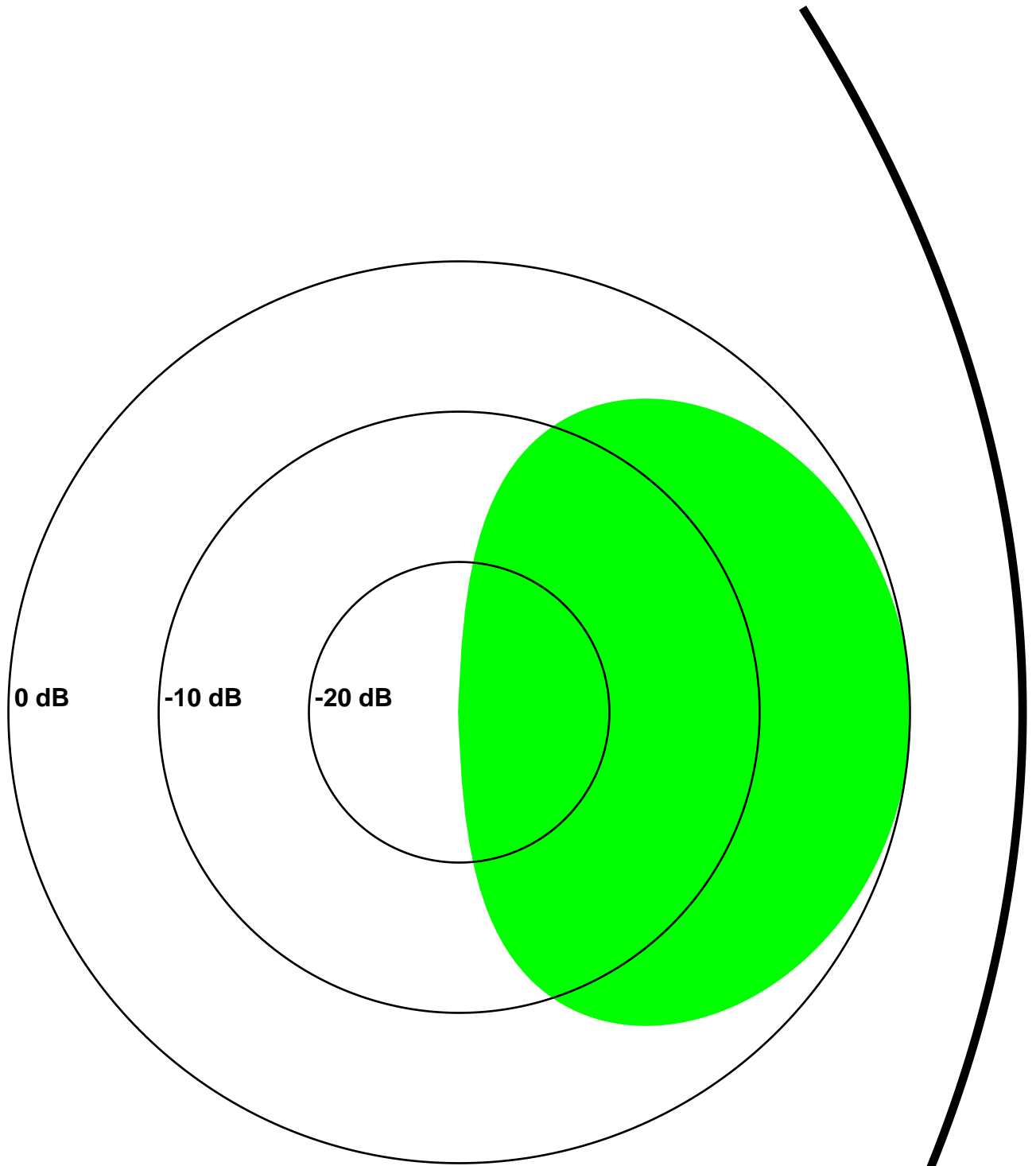
Figure 4-2

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Desired Dish Illumination - Uniform Reflector Illumination

Figure 4-3



Parabolic Dish Antenna with Typical Feed Horn Illumination

Figure 4-4

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focus is farther from the edge of the reflector than from the center. Since radiated power diminishes with the square of the distance (inverse-square law), less energy is arriving at the edge of the reflector than at the center; this is commonly called space attenuation or space taper. In order to compensate, we must provide more power at the edge of the dish than in the center by adjusting the feed pattern as shown in Figure 4-3, in order to have constant illumination over the surface of the reflector.

Simple feed antennas, like a circular horn (coffee-can feed) that many hams have used, have a pattern which can be approximated by an idealized $\cos^n \theta$ pattern like the one shown in Figure 4-4. In Figure 4-5 we superimpose the idealized pattern on our desired pattern; we have too much energy in the center, not enough at the edges, and some misses the reflector entirely. The missing energy at the edges, shown in blue, is called illumination loss and the energy that misses the reflector, shown in red, is called spillover loss. The more energy we have at the edge, the more spillover we have, but if we reduce spillover, then the outer part of the dish is not well illuminated and is not contributing to the gain. Therefore, simple horn feeds are not ideal for dish feeds (although they are useful and work very well in some applications). In order to have very efficient dish illumination we need to increase energy near the edge of the dish and have the energy drop off very quickly beyond the edge.

Edge Taper

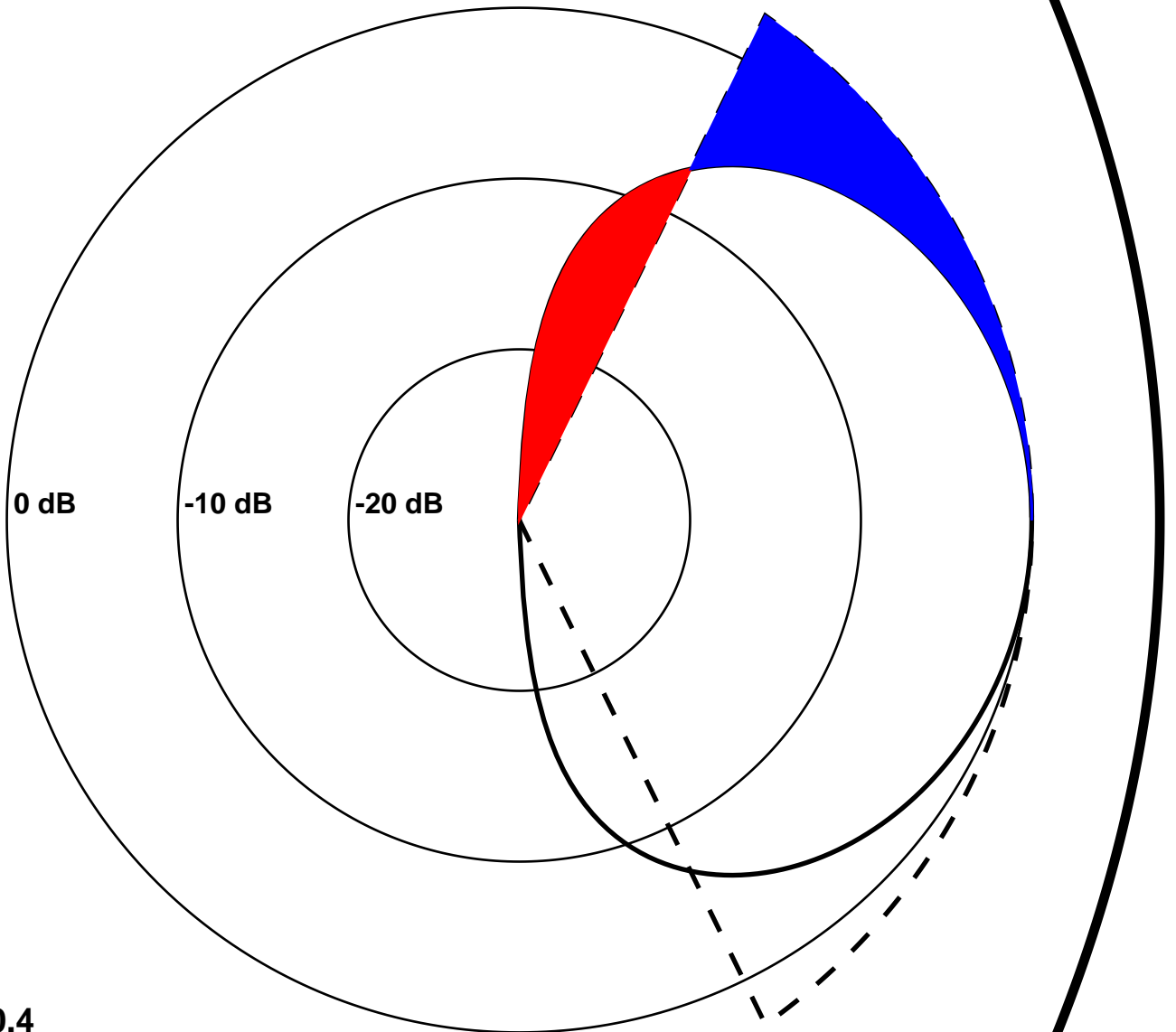
Almost all feedhorns will provide less energy at the edge of dish than at the center, like Figure 4-4. The difference in power at the edge is referred to as the edge taper. With different feedhorns, we can vary the edge taper with which a dish is illuminated. Different edge tapers produce different amounts of illumination loss and spillover loss, as shown in Figure 4-6: a small edge taper results in larger spillover loss, while a large edge taper reduces the spillover loss at the expense of increased illumination loss.

If we plot these losses^{4,6} versus the energy at the edge of the dish in Figure 4-7, we find that the total efficiency of a dish antenna peaks with an illumination taper, like Figure 4-6, so that the energy at the edge is about 10 dB lower than the energy at the center. This is often referred to as 10 dB edge taper or edge illumination — often recommended but not explained.

G/T

When an antenna is receiving a signal from space, like a satellite or EME signal, there is very little background noise emanating from the sky compared to the noise generated by the warm 300K earth during terrestrial communications. Most of the noise received by an antenna pointed at the sky is earth noise arriving through feed spillover. As we saw in Figure 4-6, the spillover can be reduced by increasing the edge taper, while Figure 4-7 shows the efficiency, and thus the gain, decreasing slowly as edge taper is increased. The best compromise is reached when **G/T**, the ratio of gain to antenna noise temperature, is maximum. This typically occurs with an edge taper of about 13 dB², but the optimum

- *Illumination loss*
- *Spillover loss*



f/D = 0.4

Illumination taper = 10 dB

Figure 4-5. Typical vs. Desired Dish Illumination

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■ *Illumination loss*
■ *Spillover loss*

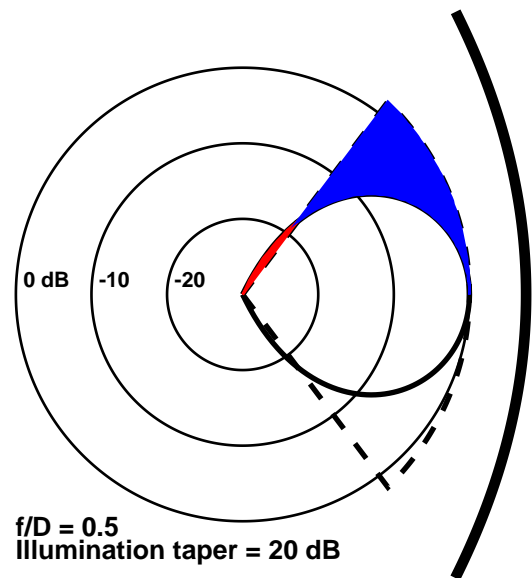
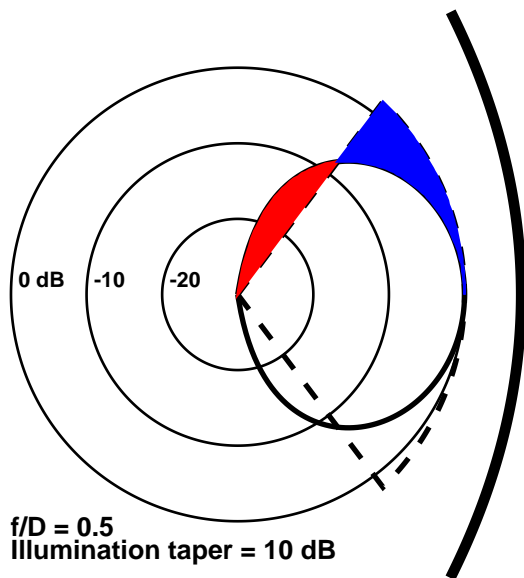
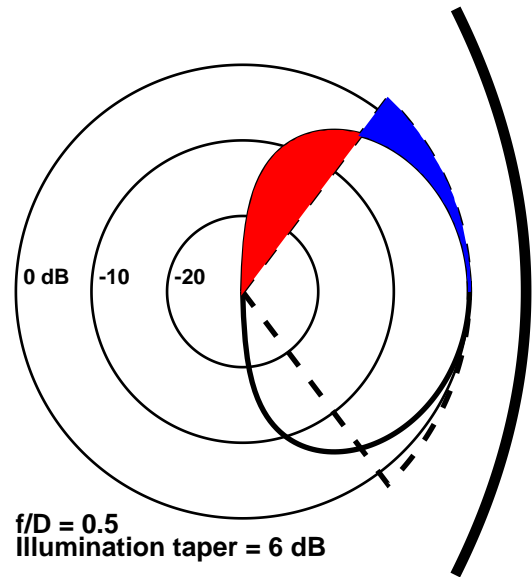
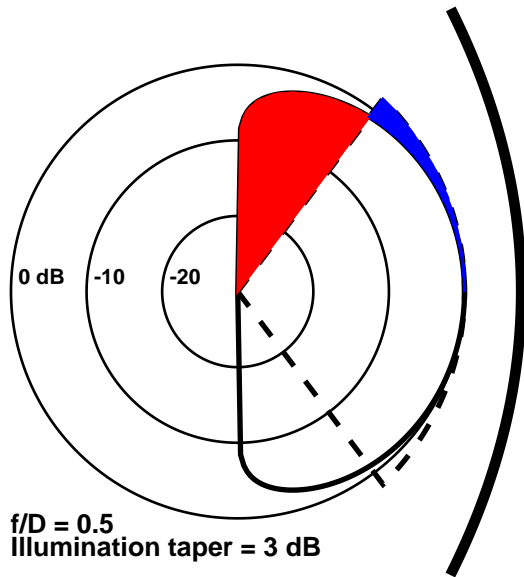


Figure 4-6. Dish Illumination with Various Illumination Tapers
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edge taper for G/T is a function of receiver noise temperature and sky noise temperature at any given frequency.

Focal Length and f/D ratio

All parabolic dishes have the same parabolic curvature, but some are shallow dishes, while others are much deeper and more like a bowl. They are just different parts of a parabola which extends to infinity. A convenient way to describe how much of the parabola is used is the f/D ratio, the ratio of the focal length f to the diameter D of the dish. All dishes with the same f/D ratio require the same feed geometry, in proportion to the diameter of the dish. The figures so far have depicted one arbitrary f/D ; Figure 4-8 shows the relative geometries for commonly used f/D ratios, typically from 0.25 to 0.65, with the desired and idealized feed patterns for each.

Notice the feedhorn patterns for the various f/D ratios in Figure 4-8. As f/D becomes smaller, the feed pattern to illuminate it becomes broader, so different feedhorns are needed to properly illuminate dishes with different f/D ratios. The feedhorn pattern must be matched to the reflector f/D . Larger f/D dishes need a feedhorn with a moderate beamwidth, while a dish with an f/D of 0.25 has the focus level with the edge of the dish, so the subtended angle that must be illuminated is 180 degrees. Also, the edge of the dish is twice as far from the focus as the center of the dish, so the desired pattern would have to be 6 dB stronger (inverse-square law) at the edge as in the center. This is an extremely difficult feed pattern to generate, and consequently, it is almost impossible to efficiently illuminate a dish this deep.

Phase Center

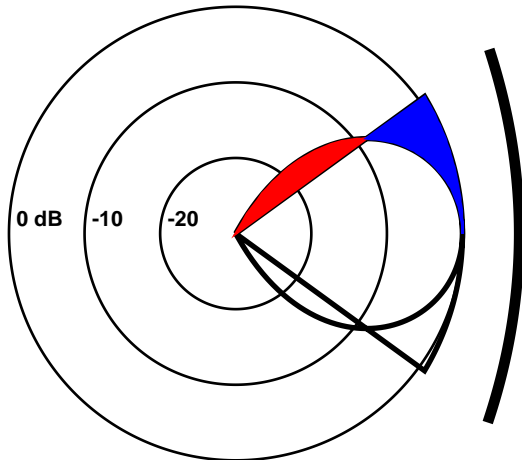
A well designed feed for a dish or lens has a single phase center, as described in Chapter 1, so that the feed radiation appears to emanate from a single point source, at least for the main beam, the part of the pattern that illuminates the dish or lens. Away from the main beam, the phase center may move around and appear as multiple points, as stray reflections and surface currents affect the radiation pattern. Also, the phase center will move with frequency, adding difficulty to broadband feed design. Fortunately, we are only considering narrow frequency ranges here.

Symmetry of E-plane and H-Plane

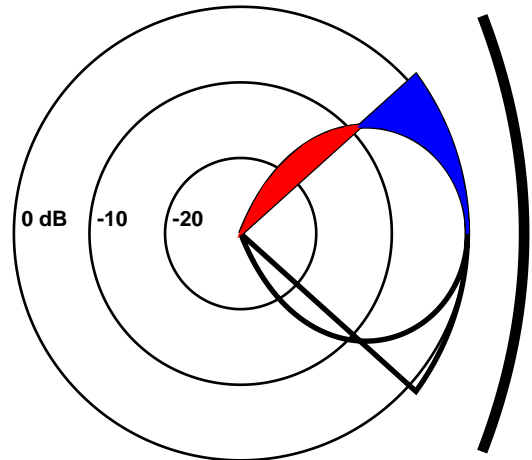
On paper, we can only depict radiation in one plane. For a simple antenna with linear polarization, like a dipole, this is all we really care about. A dish, however, is three-dimensional, so we must feed it uniformly in all planes. The usual plane for linear polarization is the **E**-plane, while the plane perpendicular to it is the **H**-plane. Unfortunately, most antennas not only have different radiation patterns in the **E**- and **H**-planes, but also have different phase centers in each plane, so both phase centers cannot be at the focus.

 *Illumination loss*

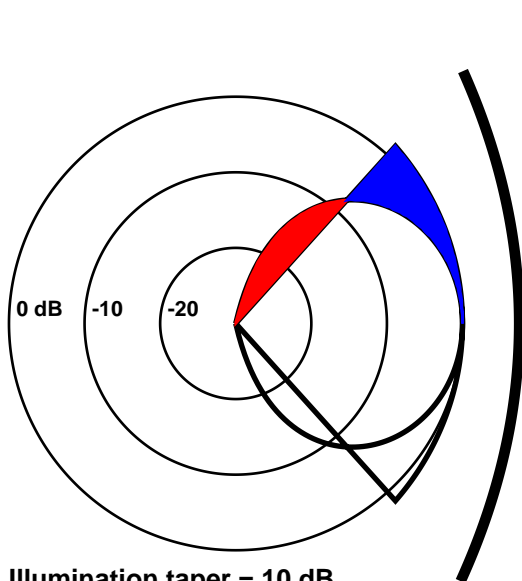
 *Spillover loss*



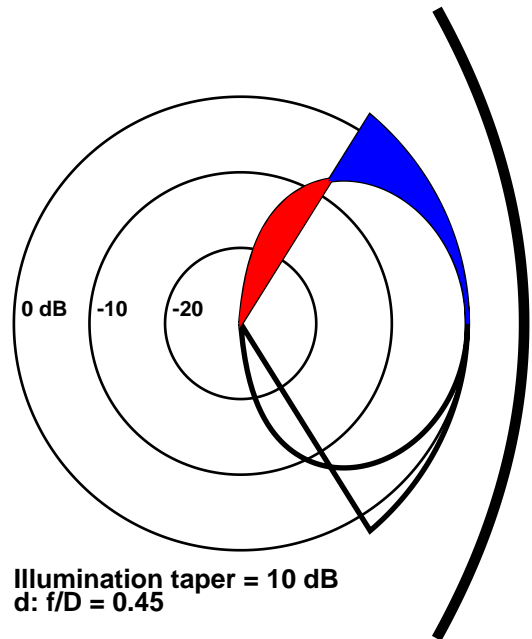
Illumination taper = 10 dB
a: $f/D = 0.75$



Illumination taper = 10 dB
b: $f/D = 0.65$



Illumination taper = 10 dB
c: $f/D = 0.55$



Illumination taper = 10 dB
d: $f/D = 0.45$

Figure 4-8. Dish Illumination for Various f/D Ratios

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- *Illumination loss*
- *Spillover loss*

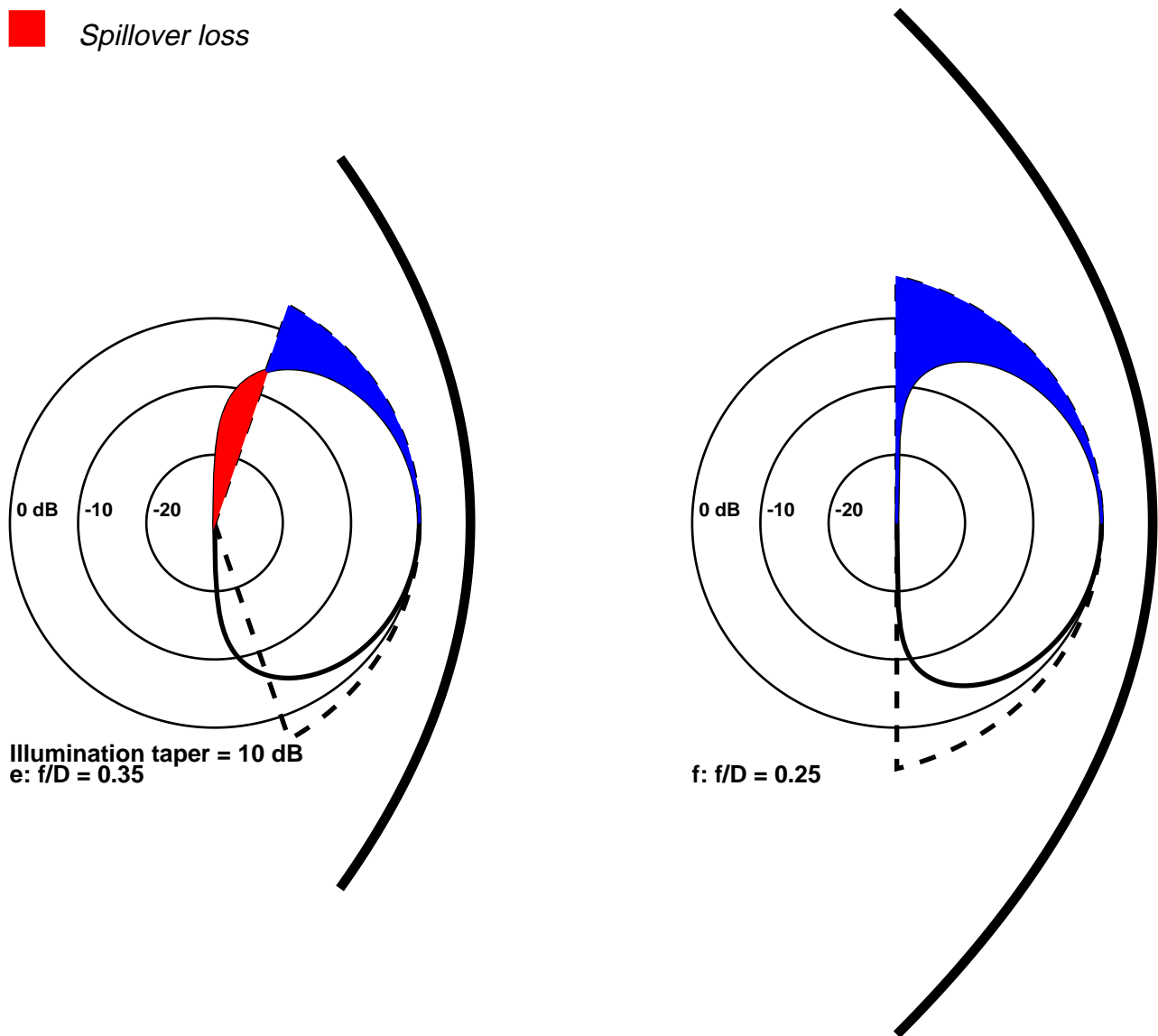


Figure 4-8 Cont. Dish Illumination for Various f/D Ratios

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Focal Length Error

When I started actually measuring the gain of dish antennas, I discovered the most critical dimension to be the focal length — the axial distance from the feed to the center of the dish. A change of 1/4 inch, or about a quarter-wavelength at 10 GHz, changed the gain by a dB or more, shown below as measured on a 22 inch dish with $f/D = 0.39$:

Measured effect of focal length error at 10 GHz:

Feed distance	Relative gain
8.125"	-0.6 dB
8.250"	0 dB
8.375"	-0.3 dB
8.625"	-1.7 dB

I was surprised at this sensitivity, since my experience with optics and photography suggested that this is not so critical — it would be extremely difficult to adjust a lens or telescope to an optical quarter wavelength. But lenses become more critical to focus as the f stop is increased — an $f/2$ lens is considered to have a very small depth of field, while an $f/16$ lens is has a large depth of field, or broad focus. The f -stop of a lens is the same as the f/D ratio of a dish — both are the ratio of the focal length to the aperture diameter. A typical reflector telescope has a parabolic reflector of $f/8$, but a dish antenna with $f/D=0.4$ has an f -stop of 0.4, so focusing is much more critical.

More reading located an article⁷ which described how to calculate the loss due to focal length error. Figure 4-9 shows the loss as the feedhorn is moved closer and farther than the focus for various f/D dishes with uniform illumination; the tapered illumination we use in practice will not have nulls as deep as the curves shown in Figure 4-9. It is clear that dishes with small f/D are much more sensitive to focal length error. Remember that a wavelength at 10 GHz is just over an inch.

The critical focal length suggests that it is crucial to have the phase center of the feed exactly at the focus of the reflector. Since the phase center is rarely specified for a feedhorn, we must determine it empirically, by finding the maximum gain on a reflector with known focal length.

If we are using a feedhorn with different phase centers in the **E**- and **H**-planes, we can also estimate the loss suffered in each plane by referring to Figure 4-9.

Lateral errors in feedhorn position are far less serious; small errors have little effect on gain, but do result in shifting the beam slightly off boresight.

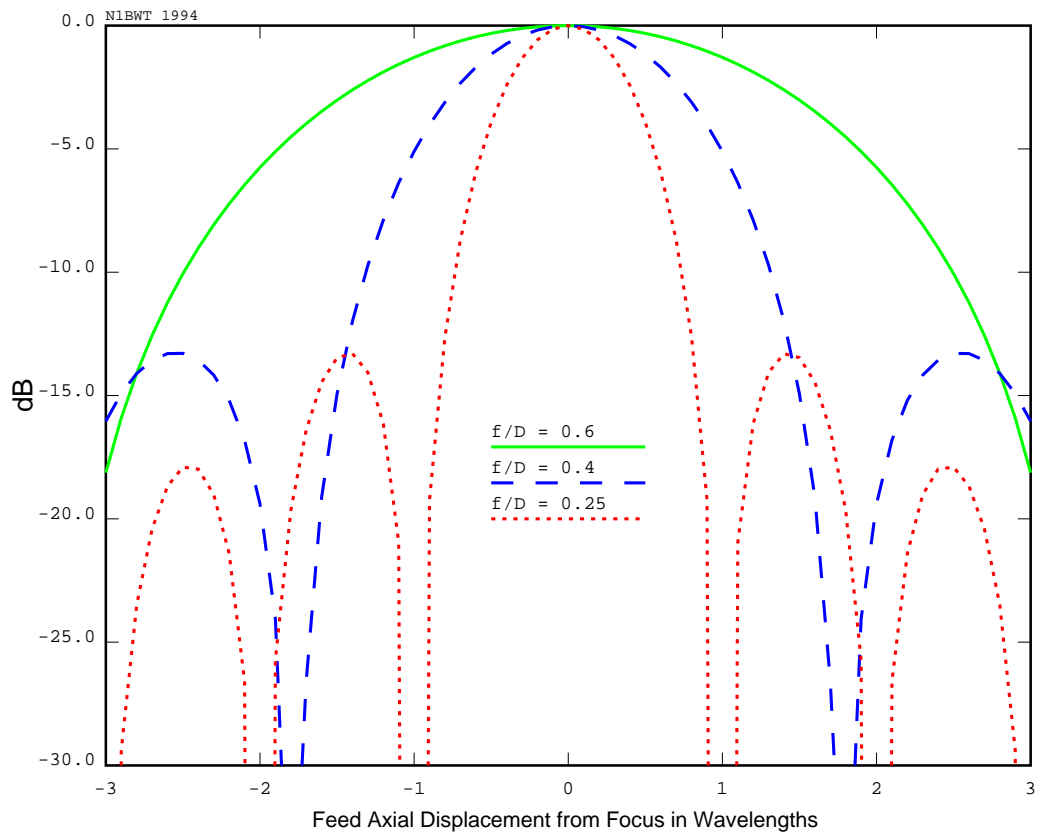


Figure 4-9. Loss due to Axial Feed Displacement from Focus

Notice that the focal length error in Figure 4-9 is in *wavelengths*, independent of the dish size. A quarter-wavelength error in focal length produces the same loss for a 150 foot dish as for a 2 foot dish, and a quarter-wavelength at 10 GHz is just over ¼ inch. Another implication is that multiband feeds, like the WA3RMX⁸ triband feed, should be optimized for the highest band, since they will be less critical at lower bands with longer wavelengths.

Total Efficiency

It has been fairly easy to calculate efficiency for an idealized feed horn pattern due to illumination taper and spillover, but there are several other factors that can significantly reduce efficiency. Because the feed horn and its supporting structures are in the beam of the dish, part of the radiation is blocked or deflected. A real feed horn also has sidelobes, so part of its radiation is in undesired directions and thus wasted. Finally, no reflector is a perfect parabola, so the focusing of the beam is not perfect. We end up with quite a list of contributions to total efficiency:

- Illumination taper
- Spillover loss
- Asymmetries in E- and H-Planes
- Focal point error
- Feedhorn sidelobes
- Blockage by the feed horn
- Blockage by supporting structures
- Imperfections in parabolic surface.
- Feedline loss

KI4VE³ suggests that the amateur is lucky to achieve 45-50% efficiency with a small dish and a typical coffee-can feed. I suspect that the only way to find total efficiency, or to optimize it, is to make gain measurements on the complete antenna.

Practical Feed Systems

An optimum feed would approximate the desired feed pattern for the f/D of the parabolic reflector in both planes and have the same phase center in both planes. We will examine actual feed horn designs in Chapter 6, and do computer analysis of some of the feeds in Chapter 11.

Complete Dish Antennas

Many of the papers describing feed horns show great detail of the horn performance, but very few even mention what happens when a reflector is added. The reflector may add too many uncertainties for good research, but our goal is to make a good working

antenna. We want high efficiency because a dish has the same size, wind loading, and narrow beamwidth regardless of efficiency — we should get as much performance as possible for these operational difficulties. In other words, if I am going to struggle with a one-meter diameter dish on a windy mountaintop, I certainly want one meter worth of performance!

The emphasis here is on smaller dishes intended for mountaintopping and other portable operation, so maximum gain with minimum size and weight is a definite consideration. For other applications, there would be other considerations; EME, for instance, would mandate maximum performance.

Parabolic Reflector

I have managed to collect a number of parabolic reflectors of various sizes and origins, and wanted to know if they were useful at 10 GHz. First, for each dish I measured the diameter **D** and the depth **d** in the center of the dish in order to calculate the focal length and **f/D** ratio. This can only be an approximation for some dishes, due to holes or flat areas in the center. The focal length is calculated as:

$$f = \frac{D^2}{16d}$$

The **HDL_ANT** computer program does the calculation and then generates a PostScript™ plot of a parabolic curve for the specified diameter and **f/D** ratio. For each reflector, I made a series of plots on a laser printer for a range of **f/D** near the calculated value, cut out templates, then fitted them to the surface to find the closest fit. For 10 GHz, the surface must be within +/- 1 mm of a true parabola⁹ for optimum performance, although errors up to +/- 3 mm result in only 1 dB degradation. I selected several reflectors with good surfaces, and discarded one that wasn't even close.

Given a choice, a reflector with a large **f/D** (0.5 to 0.6) would be preferable. As described earlier, dishes with small **f/D** are harder to illuminate efficiently, and are more sensitive to focal length errors. On the other hand, a dish that is available for the right price is always a good starting point!

Parabolic reflectors can have many sources, not just antenna manufacturers. Some aluminum snow coasters (now unfortunately replaced by plastic, but aluminum foil glued to the surface might make them usable) are good, and hams in Great Britain have put dustbin lids into service as effective parabolic reflectors for years.

Homebrewing a parabolic reflector is possible, but great difficulty is implied by the surface accuracy cited above. The surface accuracy requirement scales with wavelength, so the task is easier at lower frequencies. Of course, hams are always resourceful — N1IOL found that the cover from his 100 pound propane tank was an excellent 14 inch parabolic

surface, and has used it to mold a number of fiberglass reflectors. K1LPS then borrowed a larger cover from a different type of propane tank and found it to be nowhere near a parabola!

Mechanical support

There are two critical mechanical problems: mounting the feedhorn to the dish, and mounting the dish to the tripod. Most small dishes have no backing structure, so the thin aluminum surface is easily deformed. K1LPS discovered that some cast-aluminum frying pans have a rolled edge that sits nicely on the back of a dish; Mirro™ is one suitable brand. This is a good use for that old frying pan with the worn-out Teflon coating, so buy a new one for the kitchen. Tap a few holes in the edge of the old pan, screw the dish to it, and you have a solid backing. A solid piece of angle iron or aluminum attaches the bottom of the frying pan to the top of a tripod. The photograph in Figure 4-10 shows a dish mounted using a frying pan. WA1MBA uses this technique for a 24 inch dish at his home QTH and reports that it stands up well to New England winters.

The mounting structure for the feedhorn is in the RF field, so we must minimize the blockage it causes. We do this by keeping the support strut diameter small, by using insulating materials, and by mounting the struts diagonally, so they aren't in the plane of the polarization. Fiberglass is a good material; plant stakes or bicycle fenders are good sources, and WA5VJB recommends cheap target arrows. Use of four rather than three struts is recommended — if they are all the same length, then the feed is centered. The base of the struts should be attached to the backing structure or edge of the frying pan; the thin dish surface is not mechanically strong.

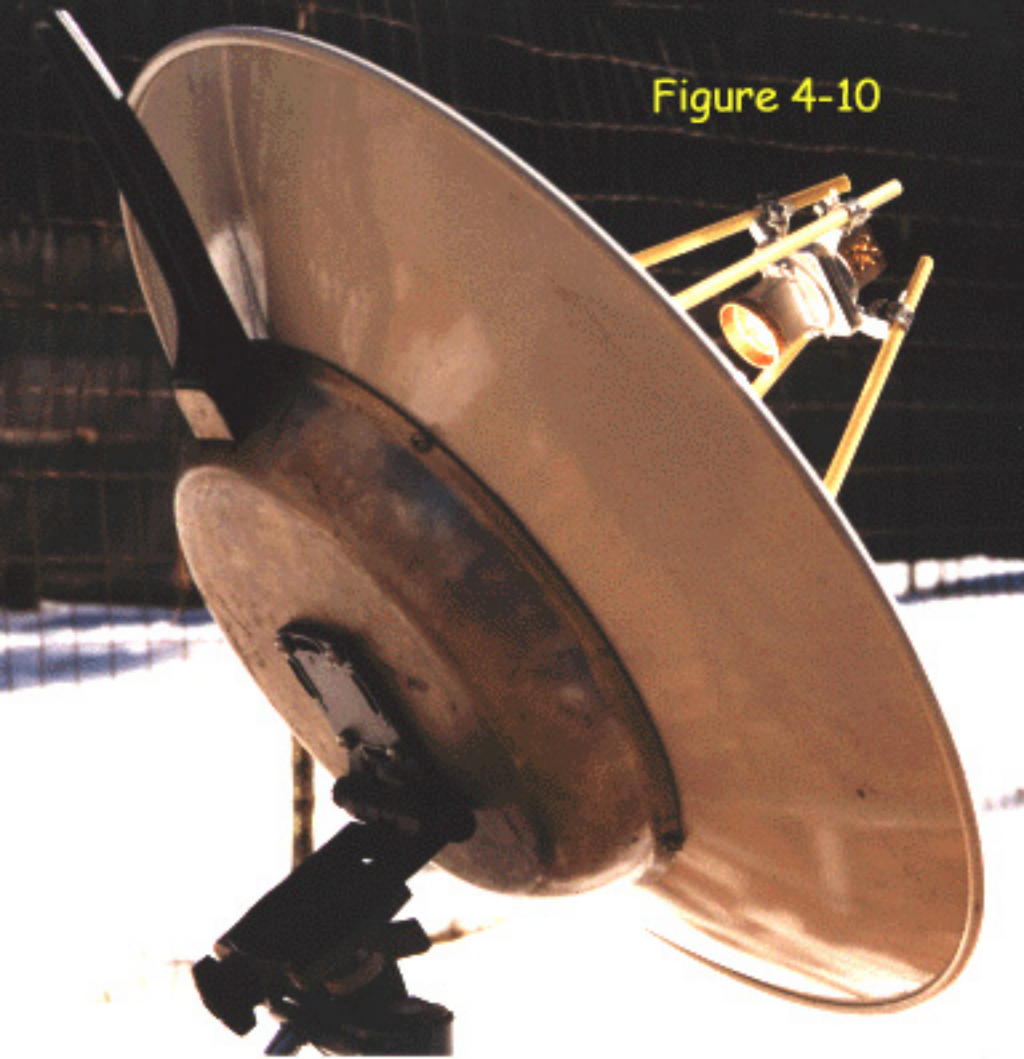
Aiming

A quality compass and a way of accurately aligning the antenna to it are essential for successful operation. Narrow beamwidth and frequency uncertainty can make searching for weak signals frustrating and time-consuming. A heavy tripod with setting circles is a good start; hang your battery from the center of the tripod and it won't blow over as often. Calibrate your headings by locating a beacon or station with a known beam heading rather than by eyeballing the dish heading; small mechanical tolerances can easily shift the beam a few degrees from the apparent boresight. As W1AIM can testify, having the wind blow a dish over can distort it enough to move the beam to an entirely different heading.

Cassegrain and Newtonian Feeds

Large professional antennas often use multiple reflector feeds, like the Cassegrain¹¹ (hyperbolic subreflector) and Gregorianian (elliptical subreflector) configurations. Even better is a shaped-reflector system¹², where both reflector shapes are calculated for best efficiency and neither reflector is parabolic; JPL reports 74.5% efficiency¹³ on their 34 meter high-efficiency antenna, and we all helped pay for it!

Figure 4-10



All of these systems require a carefully shaped subreflector which is more difficult than a parabola to fabricate. For a shaped reflector to work well, it must be larger than 10 wavelengths, and the main reflector must be much larger to minimize subreflector blockage. One analysis¹⁴ showed that a Cassegrain antenna must have a minimum diameter of 50 wavelengths with a minimum subreflector diameter of 20 wavelengths before the efficiency is higher than an equivalent dish with a primary feed. This is a fairly large dish, even at 10 GHz, and shaping a 20λ subreflector is beyond the ingenuity of most hams. However, there is probably a surplus one somewhere, and the scrounging ability of hams should never be underestimated.

Alternatives

The narrow beamwidth of a dish may actually make contacts more difficult, particularly in windy conditions. I have worked six grids from Mt. Wachusett in central Massachusetts using a small Gunnplexer horn. The longest path, 203 km., required a 12" lens¹⁰ for additional gain to make the contact on wideband FM; it would have been easy with narrowband SSB or CW.

For a rover station, a reasonable size horn might be a good compromise, with adequate gain and moderate beamwidth for easy aiming. I often use the 17.5 dBi Gunnplexer horn, with a 12" lens ready to place in front of it when signals are marginal.

Summary

A parabolic dish antenna can provide very high gain at microwave frequencies, but only with very sharp beamwidths. To achieve optimum gain, careful attention to detail is required: checking the parabolic surface accuracy with a template, matching the feedhorn to the f/D of the dish, and, most importantly, accurately locating the phase center of the feedhorn at the focus.

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